How Smooth is Space?

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Modern physics is striding towards the final theory of everything. Increasingly, the more radical notions of philosophy are beginning to find mathematical expression in the equations of particle physics. It is the fusion of physics and philosophy to which theorists are increasingly turning in the search for that holy grail, the fusion of relativity theory and quantum mechanics...

Everything happens in space. Every theory must therefore include some basic idea of what space actually is. What do you think space is? Webster's dictionary gives 23 different definitions of the word space, which are all used frequently. General relativity, the theory with the most complete treatment of the idea of space gives three concepts necessary for us to understand more clearly what space is: dimension, curvature and continuity.

Dimensionality is a mathematical concept that tells the mathematician how many variables are required to specify a point in a space uniquely. A line, therefore, has one dimension because after having picked an origin, one can uniquely specify a point in the space of this line by giving its distance from the origin. In the same way, a room has three dimensions and the world in which we live appears to have four because apart from location we also need to specify the time at which things happen.

Curvature is more intuitive than dimension. Imagine an apple. Two numbers will suffice to specify any point on its surface uniquely, hence the surface of an apple is a two dimensional space. Now make an ant walk on this apple and trace its path with a pen. You will discover that what looks like a crooked path when the ant nears the core of the apple, is in fact completely straight in the space of the apple. Clearly the apple is curved as we see it but not as the ant sees it. It is a peculiar property that four dimensions is the minimum number of dimensions whichcurvature can be measured and observed from within the space. General relativity predicts that space is more curved the closer one is to matter. Thus one would expect that the apparent position of stars changes depending on how much matter is in between the observer and the star. Sir Arthur Eddington in 1919 observed curvature for the first time when he measured the deviation in the apparent position of stars when the sun was almost in the way from the position when the sun was far away. We, therefore, live in a curved space.

Continuity is another mathematical term which means that there is an infinite number of points on the straight line connecting two points separated by a finite distance. The ramifications are that the are no minimum distances in this space, one can move as little as one likes. If one does move, no matter how much, one will always have crossed an infinite number of other points.

Having defined these three terms, one has enough information about the space to proceed with a mathematical model or physical theory. The dimensionality of our space is quite apparently four, that is three spatial dimensions and one time dimension. Curvature of our space has also been experimentally verified. Continuity, however can never be experimentally proven to be correct. That is because we can not probe smaller and smaller distances ad infinitum. The best experiment can do is to place bounds upon continuity.

Discrete Spaces

Since we cannot prove continuity, let us answer the question of what would happen if our space were not continuous, that is discrete. From the definition, this space would then possess only a finite number of points on the straight line between any two points separated by a finite distance. This means that there is a smallest distance that one can possibly travel, called a hodon. The same is true for time only that here the smallest interval is a chronon. Notice that this space is more general than the continuous one as we can retrieve it by setting the hodon and the chronon to zero length.

Any motion in this space would thus have to occur in jumps.

The fact that we do not observe any jumping on an every day basis or even in the most accurate experiments puts great limits upon the size of both the hodon and the chronon. It is likely that they have at most the size of the Planck length and time (10^{-35} meters and 10^{-44} seconds respectively). The Planck scales are those scales on which every known physical theory and discovery ceases to be valid. Physics, at the present moment, knows nothing about what happens at these scales.

One can picture the situation as a chessboard. The figures can move from square to square but they must, if they move, move in multiples of a complete square. The king may not move only half a square. The same situation would occur in our space, we would have to move from hodon to hodon in units of chronons. This immediately brings us to the first conclusion one can draw from this: There exists a smallest speed, apart from zero, that is equal to one hodon in one chronon. But could one not move one hodon in several chronons? Indeed one could not. At any instant or within any one chronon, one must be at a particular hodon. So the motion would look like: One hodon in one chronon and then the object would stop for several chronons and repeat its motion. Viewed from a great distance, this would look like a slow and continuous speed but it is not. During the one choron that the object actually moved, its speed was one hodon in one chronon. Hence if any motion occurs, it must occur with a speed equal to integer multiples of this basic speed. This is only true at the level of individual hodons. If the observer were far away, then a combination of motions and stops could yield an arbitrarily small speed over distances large compared to a hodon.

Not only speed can thus be 'quantised,' but also direction. On the chessboard, the figures can move horizontally, vertically, and diagonally. No other directions are possible in crossing just one boundary. Thus any motion must consist of these basic constituents in our space also. When looked at from a great distance, such a motion will appear to occur in a straight line and continuously but it will not actually be in a straight line nor continuous. When one wants to move at 45 degrees to an axis, one would move diagonally across the hodons. One could also move at 30 degrees to this axis, but then one would need to repeatedly move two hodons horizontally and one vertically. If the observer is far away, any directionality would disappear. However at a fundamental level, even direction is 'quantised.'

The two quantisation conditions for speed and direction are analogous to the energy quantisation required by quantum mechanics. On a fundamental level (if the observer is close to the action), the observer will see quantised energies. However, the further away the observer moves, the more smeared out this spectrum becomes. This quantised energy has is actually routinely observed in the laboratory, in contrast to the speed and direction quantisation rules above, which have never been observed.

If we have a minimum length of one hodon, everything must be at least one hodon in size. Thus there exists a smallest fundamental particle. If we are able to choose the smallest particle from the set of all fundamental particles, then by set theory, this set must be finite. It follows that there exists a biggest particle. From this finite set, there are only a finite number of permutations and combinations that one can make using only a finite number of total particles. Thus we can only have a finite number of different elements and materials. This number is radically cut by effects such as radioactive decay and other decay processes, but this does not concern our theory at the moment.

Having talked so much about hodons and chronons, how would one measure the size of one of these? Actually within the space, this distance has no meaning since anything 'in between' two hodons does not exist in this space. However, we have already noticed that we must make use of the concept of the size of a hodon and a chronon to predict anything from their existence. There is a way in which we can make sense of this.

The chessboard is within our space. Its hodon is much bigger than ours, hence we can put a number to its size. Say a chessboard hodon is 5 centimeters in length and square. Thus we know its size and can thus talk about its space. We do the same for our space. Let us take our four dimensional space and envelop or embed it in a six dimensional continuous space. Why should this be six dimensional? As it turns out, if we want to make sense of the motion of our space and other such concepts we must have one more dimension of each kind to embed the original space into. Thus this six dimensional space has four spatial dimensions and two temporal dimensions. If we do this, we can easily put a meaningful quantity to the length of a hodon in this ten dimensional superspace.

It is peculiar that we have come up with the number ten for the dimensionality of our superspace. Ten is the number of dimensions that the only current candidate for the theory of everything, namely superstring theory proposes our space to have[1].

What happens to these other dimensions though? If they were on the same footing as our discrete ones, we should just be able to jump between dimensions as we wish. One can actually compactify dimensions. That is these other dimensions would be tightly curled up and so small that one could not access them. This works like a piece of paper. It is three dimensional but its thickness is so small that we can ignore it and call a piece of paper effectively two dimensional. For our purposes, we need the extra continuous dimensions to be as big as a hodon or chronon depending on its type, that is they have to have a size of the Planck scales, which is what superstring theory also predicts. If this is the case, we could not observe or use these dimensions at all because of their small size.

Justifications for Discrete Spaces

So far we have discussed what the consequences of a discrete space would be, but we have not made the idea itself plausible apart from there being a possibility of space being indeed discrete. As long ago as 500 BC, Zeno, a Greek philosopher, proposed a series of four paradoxes[2] that make us think about space much more than we ordinarily would. One of them will suffice for the present discussion. Zeno says that if an one wishes to move from point A to B, one first needs to get to C which is midway between A and B. However to get to C, one first needs to reach D, which is midway between A and C. If one continues this argument, one sees that one needs to get to an infinite number of points before one gets anywhere. Thus no motion is possible because an infinite number of actions would have to be made in any motion, no matter how small, and this can not be done in a finite time, which is contrary to experience, of course. This paradox can be elegantly disposed of, if the space is discrete. In this case, one does not need to get to an infinite number of points before one gets anywhere because space is not infinitely subdivisible by definition. Thus, since we know that motion is possible, we have a justification, if not a proof, for the discreteness of our space.

Conclusion

I have presented my ideas for a new picture of space in which space is not continuous but discrete. It was found that this implies a quantisation of speed and direction in any motion and also implied a smallest fundamental particle in a finite set of particles. It was argued that to make the space self-consistent and meaningful, this space needed to be embedded in a six dimensional space, giving us a superspace of ten dimensions as in superstring theory. Perhaps this is one of the physical principles that will in the future lead physics towards the final theory of everything .

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References

[1] Peat, David F. Superstrings and the Search for the Theory of Everything. Contemporary Books: Chicago (1988)

[2] Sainsbury, R. M. Paradoxes Cambridge University Press: Cambridge (1988)